P425/1
PURE MATHEMATICS
Paper 1
May. 2022
3 hours

EXTERNAL MOCK EXAMINATIONS 2022 (SET 1)

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the eight questions in section* **A** *and any* **five** *questions from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All necessary working must be clearly shown.

Begin each answer on a fresh sheet of paper.

Silent, non – programmable scientific calculators and mathematical tables with a list of formulae may be used.

Neat work is a must!!

SECTION A (40 MARKS)

Answer **all** *questions in this section*.

1. Given that α and β are roots of the equation $x^2 - \mathrm{b}x + c = 0$.
Show that $(\alpha^2 + 1)(\beta^2 + 1) = (c - 1)^2 + b^2$. *(05marks)*

2. Solve the equation $3 + 2sin2\theta = 2sin\theta + 3cos^2\theta$, for $-180^0 \le \theta \le 180^0$ *(05marks)*

3. A solid right circular cylinder of height, h and radius, r has a total external surface area of $600cm^2$. Show that for maximum volume, V, the ratio $h : r = 2 : 1$. *(05marks)*

4. Find the square of the perpendicular distance from the point P with position vector $\begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}$ to the line $r = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ *(05marks)*

5. Prove by induction that $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$. *(05marks)*

6. A and B are the points $(-3, 2)$, $(5,8)$ respectively. Find the equation of the locus of a point P which moves so that $PA^2 - PB^2 = 50$. *(05marks)*

7. Evaluate $\int_0^1 \left(\frac{3-2x}{2x-1}\right) dx$ *(05marks)*

8. Solve the differential equation $x\frac{dy}{dx} = 3x - 2y$, given that $y = {}^3/_4$ when $x = 1$.
(05marks)

SECTION B (60 MARKS)

*Answer any **five** questions from this section. All questions carry equal marks.*

9. (a) Simplify: $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$ *(04marks)*

(b) Prove that $2log_c(a+b) = 2log_c a + log_c\left(1 + \frac{2b}{a} + \frac{b^2}{a^2}\right)$. *(05marks)*

(c) If $x = 2 - \sqrt{3}$, find the value of $x^2 + \frac{1}{x^2}$ *(03marks)*

10. Show that the tangents to the ellipse $x^2 + 2y^2 = 18$ at the points $(0, -3)$, $\left(-\frac{72}{17}, -\frac{3}{17}\right)$ intersect on the normal at the point $(4, 1)$. Hence find the point of intersection of the tangent and the normal. *(12marks)*

11. (a) Given that $\int_1^a \left(x + \frac{1}{2}\right) dx = 2\int_0^{\frac{\pi}{4}} sec^2 x\, dx$. Find the possible value(s) of a. *(06marks)*

(b) Show that the volume generated by the revolution about the x −axis of the area enclosed between the x −axis and the curve $cy = (x-a)(x-b)$ is

$$\frac{\pi}{30} \cdot \frac{(a-b)^5}{c^2}.$$ *(06marks)*

12. Prove that in any traingle ABC, with sides a, b, c,

(a) $\frac{1}{a}cos^2\frac{1}{2}A + \frac{1}{b}cos^2\frac{1}{2}B + \frac{1}{c}cos^2\frac{1}{2}C = \frac{(a+b+c)^2}{4abc}$ *(06marks)*

(b) $sin\frac{1}{2}(B - C) = \left(\frac{b-c}{a}\right)cos\frac{1}{2}A$ *(06marks)*

13. Given that $f(x) = \frac{x^2 + 3x}{x + k}$. Find the range of values k can take for real x.

Sketch $f(x)$ for $k = -1$ *(12marks)*

14. The planes A and B are given by the equations $3x + 2y + z = 4$ and $2x + 3y + z = 5$ respectively. The plane C containing the point $P(2, 2, 1)$ is perpendicular to each of the planes A and B. Find

(a) the distance from the point P to the plane A. *(06marks)*

(b) cartesian equation for the line of intersection of the planes A and B. *(03marks)*

(c) a cartesian equation for the plane, C. *(03marks)*

15. (a) Find the three numbers in geometrical Progression such that their sum is 39 and their product is 729 *(07marks)*

(b) Expand $\left(1 - \frac{3}{2}x - x^2\right)^5$ in ascending powers of x as far as the term in x^3 *(05marks)*

16. The number of Covid – 19 victims in a given country will grow at a rate that is proportional to the current number of victims. In the absence of any outside factors, the number of victims will triple in two weeks time. On any given day there is a net migration into the community of 15 victims and 16 are put under quarantine by Health Ministry and 7 die.

(a) If there are initially 100 victims in the community, form a differential equation relating the number of victims,x at any time, t, and t . *(07marks)*

(b) If no victim survives, show that it takes approximately 7.2 weeks for them to die out. *(05marks)*

END